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Singularities in wavy strings

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Abstract: Extremal six-dimensional black string solutions with some non-trivial momentum distribution along the wave are considered. These solutions were recently shown to contain a singularity at the would-be position of the event horizon. In the black string geometry, all curvature invariants are finite at the horizon. It is shown that if the effects of infalling matter are included, there are curvature invariants which diverge there. This implies that quantum corrections will be important at the would-be horizon. The effect of this singularity on test strings is also considered, and it is shown that it leads to a divergent excitation of the string. The quantum corrections will therefore be important for test objects.

Keywords: p-branes, Black Holes in String Theory, Black Holes.

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1. Introduction

In the last two years, sensational progress in the understanding of the thermodynamics of black holes has been achieved. A statistical understanding of the Bekenstein-Hawking entropy for a large number of black holes has been obtained. That is, it is possible to find states in string theory at weak coupling whose strong coupling limit corresponds to a black hole, and the number of states that correspond to a particular black hole is given by the exponential of the black hole's entropy (for reviews, see [1, 2]). The first successful counting was obtained for extreme five-dimensional black holes [3]. One can think of these as extreme black strings in six dimensions, with the direction along the string compacitified. One way to generalize this solution is by adding traveling waves moving along the string, i.e., by making the momentum distribution along the string inhomogeneous [4, 5]. In [6, 7], it was shown that the statistical understanding of the Bekenstein-Hawking entropy can be extended to these black strings with traveling waves. This is an impressive success for the string picture, as the entropy depends on arbitrary functions describing the wave.

This success seems somewhat mysterious, in light of the later observation [8, 9] that the would-be event horizon of the black strings with traveling waves is singular. However, this singularity appears to be a fairly mild one. The total tidal distortion of an infalling body remains finite. In a thermal ensemble of black strings with a given total momentum, the typical string will have only small inhomogeneities in the momentum distribution. For such a typical black string, the total tidal distortion only differs from that occasioned by a string without traveling waves by a factor of order one [10]. Furthermore, the Ricci tensor and all the curvature invariants remain finite at the singularity, which was used in [8] to argue that α' corrections to this solution would be suppressed.

This paper presents a further investigation of the nature of this singularity. This investigation is similar to, and inspired by, the analysis in [11] of the large curvatures near the horizon of certain near-extreme black holes. There are important differences in the details of the analysis, but the conclusions are qualitatively similar to those of [11]. I will first show that if we consider the effects of infalling matter on the geometry, invariants which diverge as the singularity at the would-be horizon is approached can be constructed. Thus, it seems likely that if one consistently incorporates the back-reaction of infalling matter, the α' corrections and perturbative quantum gravity corrections become important. This is similar to the result of [11], but in the present case, one can construct a divergent invariant as soon as there is any amount of infalling matter. This result applies for any type of matter.

The experience of [11] indicates that the existence of these divergent invariants will not fundamentally alter the motion of test particles. One might therefore wonder if the singularity is still mild in a certain sense, even if quantum corrections are important. However, in string theory, the appropriate test objects are not particles following geodesics, but rather first-quantized test strings [12]. In section 4, I show that from the point of view of a family of infalling observers, the metric looks approximately like that of a plane wave. Thus, the propagation of test strings can be approximated by the propagation in a plane wave. The expectation value of the mass squared operator $\langle M_>^2 \rangle$ diverges on propagating through the singularity at the horizon. Although I only discuss the six-dimensional black string with longitudinal waves, similar conclusions should hold for the string with internal waves, and for the five-dimensional black string. Therefore, at least for strings, the classical analysis of the motion of test objects will break down as we approach the would-be horizon, and quantum corrections will have an important effect.

The singularity at the would-be horizon is, from these two points of view, far from mild. This makes it seem even more mysterious that one can successfully reproduce the entropy of these objects using string theory. Even though the geometry near the horizon will receive large quantum corrections, it gives the correct density of states. On the other hand, the existence of large quantum corrections near the event horizon should have important consequences for the information loss problem in the context of these solutions, particularly as these corrections have an important effect on infalling observers. These solutions thus provide a new perspective on both the relation between geometry and entropy, and the information loss problem. The most important caveat is that this discussion only covers the extreme solutions, and it is not clear that this analysis can be extended to non-extreme configurations.

In the next section, I will review the six-dimensional black string solution, following [8] closely. The curvature near the would-be horizon has a form which is reminiscent of a plane wave. In section 3, I show that combining the stress-energy of infalling matter with the curvature, one can obtain invariants which diverge at the singularity. In section 4, I show that the metric can be approximated by a plane wave from the point of view of a suitable family of observers. On propagating through such a plane wave, the expectation value of the mass squared for a test string diverges. Section 5 contains some speculations on the implications of these results.

2. Black strings with traveling waves

The solutions considered here are solutions of the low-energy effective theory obtained from type IIB string theory. The solutions are expressed in the ten dimensional Einstein frame. Since only the metric, dilaton, and RR three-form H are non-trivial, they are solutions of the effective action

$$S = \frac{1}{16\pi G}\int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12}e^{\phi}H^2\right]. \tag{2.1}$$

A class of six-dimensional extremal black string solutions of (2.1) was studied in [8]. For the sake of simplicity, I will only consider solutions with a longitudinal wave and no internal wave. This allows us to reduce the solution to the six-dimensional metric

$$ds^2 = -\left(1 - \frac{r_0^2}{r^2}\right)dudv + \frac{p(u)}{r^2}du^2 + \left(1 - \frac{r_0^2}{r^2}\right)^{-2}dr^2 + r^2 d\Omega_3\,. \tag{2.2}$$

Here $v, u = t \pm z$, where z is a coordinate on an S^1 with period L, so $p(u)$ is a periodic function of u. For this solution, $\phi = 0$, so the Einstein and string frames are identical.

At the horizon $r = r_0$, this coordinate system breaks down, as $t \to \infty$ at the horizon, implying $u, v \to \infty$. A coordinate system in which the metric is C^0 at the horizon was found in [6]. This form of the metric is written in terms of a new function $\sigma(u)$, defined by

$$\sigma^2(u) + \dot{\sigma}(u) = \frac{p(u)}{r_0^4}\,, \tag{2.3}$$

which implies $\sigma(u)$ is a periodic function with the same period L as $p(u)$. We also define $R = r/r_0$,

$$G(u) = e^{\int_0^u \sigma du}\,, \tag{2.4}$$

and

$$W = G\left(1 - \frac{1}{R^2}\right)^{1/2}. \tag{2.5}$$

The necessary coordinate transformations are

$$U = -\int_u^{+\infty}\frac{du}{G^2}\,, \tag{2.6}$$

$$q = -\frac{r_0}{2W^2} - 3r_0\int_0^U \sigma dU\,, \tag{2.7}$$

$$V = v - \frac{r_0^2\sigma}{R^2 - 1} - 2r_0^2\int_0^u \sigma^2 du + 3r_0^2\int_0^U \sigma^2 W^2 dU\,. \tag{2.8}$$

The metric (2.2) can be written in terms of these coordinates as

$$\begin{aligned} ds^2 = &-W^2 dU dV + r_0^2\sigma^2 W^4 (R^2-1)(4R^2-3) dU^2 \\ &+\left[2r_0\sigma W^4 R^2 (R^2-1)(2R^2+1) + 6r_0 W^2 \int_0^U \sigma^2 W^4 dU\right] dq dU \\ &+W^4 R^6 dq^2 + R^2 r_0^2 d\Omega_3 \,. \end{aligned} \tag{2.9}$$

In this coordinate system, the future event horizon lies at $U = 0$, and all the other coordinates are well-behaved at the event horizon. The region outside the horizon corresponds to $U < 0$. The metric is independent of V; that is, $\partial/\partial V$ is a null Killing vector. The horizon area is $A = 2\pi^2 r_0^4 \int_0^L \sigma du$.

To write down the curvature components, [8] used a null sechsbein[1]

$$\begin{aligned} (e_1)_a &= Rr_0 \partial_a \theta, \quad (e_2)_a = Rr_0 \sin\theta \partial_a \phi, \quad (e_3)_a = Rr_0 \sin\theta \sin\phi \partial_a \psi, \\ (e_4)_a &= W^2 R^3 \partial_a q, \quad (e_6)_a = W^2 \partial_a U, \\ (e_5)_a &= -\frac{1}{2}\partial_a V + \frac{1}{2}\sigma^2 W^2 (R^2-1)(4R^2-3)\partial_a U \\ &\quad + \left[r_0 \sigma W^2 R^2 (R^2-1)(2R^2+1) + 3r_0 \int_0^U \sigma^2 W^4 dU\right] \partial_a q \,. \end{aligned} \tag{2.10}$$

This is a natural choice of basis, given the metric (2.9). The angles θ, ϕ, ψ are coordinates on the three-sphere, and the metric in this basis is $ds^2 = (e_1)^2 + (e_2)^2 + (e_3)^2 + (e_4)^2 + 2(e_5)(e_6)$. Some of the curvature components diverge at the horizon. The divergent terms are

$$R_{1515} = R_{2525} = R_{3535} = \frac{\dot\sigma}{R^2-1} + \text{finite part}\,, \tag{2.11}$$

and

$$R_{4545} = -\frac{3\dot\sigma}{R^2-1} + \text{finite part}\,, \tag{2.12}$$

where an overdot denotes $\partial/\partial u$. The other non-zero curvature components $\sim 1/r_0^2$ or $\sim \sigma/r_0$ near the horizon. Note that the Ricci tensor will be finite; that is, the divergent contribution to the curvature comes entirely from the Weyl tensor. This divergence in the curvature can be thought of as arising because σ is periodic in u, so it oscillates an infinite number of times as the horizon is approached.

If one writes a gravitational plane wave in a null basis, the non-zero curvature components are just R_{i5i5} (no sum on i), and the Ricci tensor vanishes. Hence, the divergent part of the curvature here is reminiscent of a gravitational plane wave. It was also shown in [8] that near $U = 0$, the curvature $\sim 1/U$. We will see in section 4 that the metric (2.9) can indeed be approximated by a plane wave near $U = 0$.

[1]This is the same notation as used in [8], even though the labels 5 and 6 appear to be reversed relative to (2.10) of [8]. This is because they are actually writing the $(e^\mu)_a$ rather than $(e_\mu)_a$.

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3. Effects of infalling matter on the geometry

In [8], it was argued that the absence of divergent curvature invariants at the would-be horizon in the black string metric might indicate that corrections to the classical geometry were suppressed. In [11], it was realized that in metrics with large null curvatures, the back-reaction of infalling matter could make an important difference in such considerations. One can construct large curvature invariants by including the matter stress tensor, even if the stress tensor remains small. In the six-dimensional black string metric, the Ricci tensor in the basis (2.10) is

$$R_{11} = R_{22} = R_{33} = -R_{44} = -R_{56} = \frac{2}{r_0^2 R^6}, \qquad R_{55} = O(\dot{\sigma}, \sigma)\,. \tag{3.1}$$

Note that in particular $R_{66} = 0$. Since the Ricci tensor doesn't diverge, we can't find large curvature invariants by considering $T_{\mu\nu}T^{\mu\nu}$, as was done in [11]. However,

$$R^{\mu\nu} R_{\mu 5 \nu 5} = \frac{12\dot{\sigma}}{r_0^2 R^6 (R^2 - 1)} + \cdots\,, \tag{3.2}$$

so $T^{\rho\sigma} R^{\mu\nu} R_{\mu\rho\nu\sigma}$ will diverge at the horizon if the stress tensor of the matter $T_{\rho\sigma}$ contains an ingoing part, that is, if $T_{66} \neq 0$. Adding any non-zero amount of ingoing matter gives rise to a divergent invariant. The presence of this divergence is also independent of the type of matter we use, and of the size of the inhomogeneities on the black string. This argument extends immediately to the five-dimensional black string, and to the black string with internal waves, as in all these cases the analogue of (3.2) diverges.

In [11], it was found that the presence of such large modifications of the invariants did not necessarily imply that the motion of infalling particles in the exact metric would deviate significantly from that in the original metric. That is, the existence of these large invariants doesn't invalidate the arguments that the total tidal distortion of infalling bodies remains finite. Essentially, this is because the infalling bodies already feel the large curvature; their tidal distortion is finite simply because the double integral of the curvature over proper time is finite. Unfortunately, the complicated nature of the black string metric precludes any analysis along the lines of [11] of the motion of infalling matter in the exact classical solution.

One might object that the divergent object constructed above is not, technically, a curvature invariant. However, even though it's too difficult to construct an exact classical solution including the curvature due to the infalling matter, I expect that it would contain divergent curvature invariants arising from similar contractions between the curvature due to infalling matter and the black hole's curvature.

The presence of large invariants presumably implies that string α' corrections and perturbative quantum corrections are becoming important. Since one obtains large invariants as soon as any infalling matter is added to the solution, such corrections must in some sense already be important in the original solution, since quantum fluctuations in the matter fields should be able to produce the necessary ingoing flux. However, the infalling observers experience small integrated tidal distortions if the inhomogeneities on the black string are small enough. One might then argue that the quantum corrections will have little effect on them.

4. Effects on test strings

In this section, we consider the behavior of test strings in the singular black string solution. A test string propagating through the would-be horizon will receive a divergent excitation. Therefore, the quantum corrections to the metric near the horizon must have an important effect on these test strings. To simplify the analysis, I show that the part of the spacetime (2.9) traversed by small infalling observers can be approximated by a plane wave metric. The approximation of the metric near the horizon was also discussed in [10].

We want to find a simpler metric which describes the geometry seen by a suitable family of infalling geodesics. There are not enough conserved quantities to allow us to determine the geodesics explicitly. However, q is a good coordinate at the horizon, so we can choose a family of observers with q approximately constant near the horizon. For such a family of observers, we can neglect the dependence on q in the metric. We also wish to neglect the dU^2 and $dqdU$ terms in the metric. These terms do not have any effect on the divergent part of the curvature; indeed, the only part of the curvature they contribute to is the sub-leading part of the R_{i5i5} terms. We might therefore argue for neglecting them on the grounds that they have no relevance for the physics we are interested in. Another weak argument for neglecting the $dqdU$ term is that it vanishes to leading order, and since q is approximately constant, the sub-leading part is less important than the $dUdV$ term. I will give an argument for neglecting the dU^2 term later in this section. The resulting approximate metric will be

$$ds^2 \approx -W^2 dUdV + W^4 R^6 dq^2 + R^2 r_0^2 d\Omega_3 \,, \tag{4.1}$$

where W and R are now just functions of U. Note that in the metric (4.1), $p_q = W^4 R^6 \dot{q}$ is a constant of motion, so there are geodesics of this approximate metric with q constant.

If we take $r_0 \gg 1$, we can also approximate the three-sphere metric $r_0^2 d\Omega_3$ by a flat metric. I will use x_1, x_2, x_3 to denote these flat directions. We should also take $r_0 \gg \sigma$ to ensure that the finite part of the curvature is negligible. Let us also define $\tilde{U}$ such that $d\tilde{U} = W^2 dU$. The metric is then

$$ds^2 \approx -d\tilde{U}dV + W^4 R^6 dq^2 + R^2 dx_i dx^i \,. \tag{4.2}$$

This is a plane wave metric. To bring it into the form used in [12], use a change of coordinates discussed in [13],

$$\tilde{V} = V + \frac{1}{2}(W^4 R^6)' q^2 + \frac{1}{2}(R^2)' x_i x^i \,, \tag{4.3}$$

$$X_i = Rx_i, \qquad X_4 = W^2R^3q\,, \tag{4.4}$$

where a prime denotes $\partial/\partial\tilde{U}$. In terms of these coordinates, the metric (4.2) is

$$ds^2 \approx -d\tilde{U}d\tilde{V} + dX_\mu dX^\mu + \left[\frac{R''}{R}X_i^2 + \frac{(W^2R^3)''}{W^2R^3}X_4^2\right]d\tilde{U}^2\,, \tag{4.5}$$

where μ runs over $1,\ldots,4$, and i runs over $1,\ldots,3$. If we had kept the dU^2 term from (2.2), we could now give an argument for neglecting it, since it vanishes at the horizon, and is therefore negligible compared to the $d\tilde{U}^2$ term in (4.5).

The geodesics in this metric have a conserved momentum $P = d\tilde{U}/d\tau$ associated with the Killing vector $\partial/\partial V$, where τ is the proper time along the geodesic. Using $d\tilde{U} = W^2dU$ and (2.6), we can write

$$P = \frac{d\tilde{U}}{d\tau} = W^2\frac{dU}{d\tau} = \frac{W^2}{G^2}\frac{du}{d\tau} = \left(1-\frac{1}{R^2}\right)\frac{du}{d\tau} = p_v\,, \tag{4.6}$$

where p_v is the momentum associated with the Killing vector $\partial/\partial v$ of the original form of the metric (2.2). For geodesics which start from infinity with small initial velocity, $|P| = |p_v| \approx 1$. Hence $\tilde{U}$ is approximately equal to the proper time along such geodesics. Note, however, that I have not shown that there are geodesics which both start from infinity with small velocity and cross the horizon with q approximately constant. I won't make any particular assumption about the size of P in the subsequent analysis.

The metric is approximately a plane wave metric. We should now consider the form of the coefficient of $d\tilde{U}^2$. Using $d\tilde{U} = W^2dU$ and (2.6),

$$\frac{R''}{R} = \frac{1}{R}\frac{\partial}{\partial\tilde{U}}\left(\frac{\partial}{\partial\tilde{U}}R\right) = \frac{G^2}{RW^2}\frac{\partial}{\partial u}\left(\frac{G^2}{W^2}\frac{\partial}{\partial u}R\right). \tag{4.7}$$

From (2.5), it follows that

$$\dot{R} = \frac{WR^3}{G^2}\dot{W} - \frac{W^2R^3}{G^3}\dot{G}\,, \tag{4.8}$$

and from (2.7) it follows that

$$\dot{W} = \frac{3\sigma W^3}{G^2}\,, \tag{4.9}$$

while $\dot{G} = \sigma G$ (here, as before, an overdot denotes $\partial/\partial u$). Substituting (4.8) and (4.9) into (4.7), we find

$$\frac{R''}{R} = \frac{G^2}{RW^2}\frac{\partial}{\partial u}\left(-\sigma R^3 + \frac{3W^2\sigma R^3}{G^2}\right) = -\frac{\dot{\sigma}}{R^2-1} + \text{finite terms}\,. \tag{4.10}$$

Similarly,

$$\frac{\partial}{\partial u}(W^2R^3) = \frac{3\sigma W^4R^3}{G^2} + \frac{6W^6\sigma R^5}{G^4}, \tag{4.11}$$

and hence

$$\frac{(W^2R^3)''}{W^2R^3} = \frac{G^2}{R^3W^4}\frac{\partial}{\partial u}\left[\frac{G^2}{W^2}\frac{\partial}{\partial u}(W^2R^3)\right] = \frac{3\dot{\sigma}}{R^2-1} + \text{finite terms}\,. \tag{4.12}$$

Note that no approximations were used in obtaining (4.10), (4.12). We see that the singular terms now appear directly in the metric. It may not seem like progress to have rewritten the metric in a form which is singular at $\tilde{U} = 0$, but this is an essential step in simplifying the task of studying infalling strings.

We can rewrite this singular term in a more useful form by following an argument given in [8]. Define

$$\sigma_0 = \frac{1}{L}\int_0^L \sigma du, \quad G_0(u) = e^{\sigma_0 u}, \quad U_0(u) = -\frac{1}{2\sigma_0 G_0^2}\,, \tag{4.13}$$

and

$$\eta_G = \frac{G}{G_0}, \qquad \eta_U = \frac{U}{U_0}\,. \tag{4.14}$$

It follows that η_G is a periodic function of u with period L, and therefore is bounded from above and below. This implies that η_U is also bounded from above and below. The divergent term can be rewritten in terms of these functions,

$$\frac{\dot{\sigma}}{R^2-1} = \frac{\dot{\sigma}G^2}{R^2W^2} = -\frac{\dot{\sigma}\eta_G^2\eta_U}{2\sigma_0 R^2W^2}\frac{1}{U} \approx -\frac{\dot{\sigma}\eta_G^2\eta_U}{2\sigma_0}\frac{1}{\tilde{U}}\,. \tag{4.15}$$

In the final step, we have taken $R \approx 1$ and W approximately constant. Thus, we see that this divergent term $\sim 1/\tilde{U}$. Let's write

$$\delta = -\frac{\dot{\sigma}\eta_G^2\eta_U}{2\sigma_0}\,. \tag{4.16}$$

This coefficient is oscillating very rapidly near the horizon, as part of it is periodic in u. At small values of $\tilde{U}$, one period of u occupies $\Delta\tilde{U} < \tilde{U}$.

To simplify the analysis of the motion of test strings, we will restrict to small inhomogeneities, i.e., δ small. This can be motivated by considering a thermal ensemble of black strings with a given total momentum. In such an ensemble, the typical state will only have small inhomogeneities in the momentum distribution. More precisely, $\dot{\sigma}_{rms} \sim \sigma_0^2/r_0^2$ [10], which implies $\eta_G^2\eta_U \sim 1$, and $\delta_{rms} \sim \sigma_0/r_0^2$. Thus, δ will be small if we have chosen the background curvatures to be small.

The metric simplifies to

$$ds^2 \approx -d\tilde{U}d\tilde{V} + dX_\mu dX^\mu + \frac{\delta}{\tilde{U}}(3X_4^2 - X_iX^i)d\tilde{U}^2\,. \tag{4.17}$$

This metric describes a singular gravitational plane wave. Let us briefly review the various approximations that have gone into this result. Taking the three-sphere directions to be flat, and ignoring the other finite contributions to the curvature, will be a good approximation so long as our test objects are small compared to r_0 and $\sqrt{r_0/\sigma}$. Taking q to be approximately constant is valid sufficiently close to the horizon, and Marolf has argued [10] that when the inhomogeneities are small, it is valid for the whole region of large curvatures.

We consider a string propagating in the metric (4.17). We quantize the string in light-cone gauge, $\tilde{U} = P\tau$, where τ is worldsheet proper time. We will work in string units, so $\alpha' = 1$. If we decompose the transverse coordinates into modes,

$$X^\mu(\sigma, \tau) = \sum_n X^\mu_n(\tau) e^{in\sigma}\,, \tag{4.18}$$

the worldsheet field equations become

$$\ddot{X}^i_n + n^2 X^i_n + \frac{\delta P^2}{\tilde{U}} X^i_n = 0 \tag{4.19}$$

and

$$\ddot{X}^4_n + n^2 X^4_n - \frac{3\delta P^2}{\tilde{U}} X^4_n = 0\,, \tag{4.20}$$

where an overdot now denotes $d/d\tau$. For $\tilde{U} \gg 0$ and $\tilde{U} \ll 0$, the metric (4.17) is approximately flat, and the solutions reduce to combinations of the usual flat space solutions $e^{\pm in\tau}$.[2] We will denote the region $U \ll 0$ by a subscript $<$ and the region $U \gg 0$ by a subscript $>$. We can define two complete sets of solutions of (4.19), (4.20). The solutions $u_{n<}, \tilde{u}_{n<}$ are defined to be pure positive and negative frequency for $U \ll 0$, while $u_{n>}, \tilde{u}_{n>}$ are defined to be pure positive and negative frequency for $U \gg 0$. There is a linear relation between these two sets of states, which implies a transformation between the initial and final mode creation and annihilation operators, the Bogoliubov transformation

$$a^i_{n>} = A_n a^i_{n<} - B^*_n \tilde{a}^{i\dagger}_{n<}, \qquad \tilde{a}^i_{n>} = A_n \tilde{a}^i_{n<} - B^*_n a^{i\dagger}_{n<}\,. \tag{4.21}$$

A string initially in the vacuum state will become excited on passing through the wave. The excitation in a particular mode is given by

$$\langle N_n \rangle = \langle 0_< | N_{n>} | 0_< \rangle = |B_n|^2\,. \tag{4.22}$$

To find the coefficients B_n, we propagate a positive-frequency solution $u_{n<}$ through the plane wave and ask for its negative-frequency part $\tilde{u}_{n>}$. The effective potential in (4.19), (4.20) is far too complicated to allow us to solve this problem exactly. However, if we consider large n, $\delta P^2/\tilde{U} \ll n^2$ except for a very narrow region. Furthermore, in this narrow region, δ is oscillating with a wavelength much shorter than the wavelength of the mode. The area of a half-cycle of this oscillation is given by

$$\int \frac{\dot{\sigma}}{R^2 - 1} d\tilde{U} = \int \frac{\dot{\sigma} G^2}{R^2 W^2} d\tilde{U} = \int \frac{\dot{\sigma}}{R^2} du\,. \tag{4.23}$$

[2] Note that for $r_0 \gg 1$, there is a region inside the horizon where it is sensible to treat the exact metric as approximately flat. I'm assuming that in the exact metric (2.9), it is possible to propagate the string through the singularity.

Thus, the areas of the half-cycles are roughly equal near the horizon. We can therefore approximate the potential by a series of closely-spaced delta functions of alternating sign. The propagation through one such delta function gives a contribution to B_n of order $1/n$, but the contribution from two successive delta functions cancels to leading order. Therefore, for the purpose of obtaining an estimate of B_n for large n, we can ignore this central part of the potential and treat the rest by the Born approximation. This gives [12, 14]

$$B_n \sim \frac{P}{2in} \int d\tilde{U} e^{-in\tilde{U}/P} \frac{\delta}{\tilde{U}} \sim \frac{P\delta_{rms}}{2in} \int dz e^{-iz} \frac{\delta}{\delta_{rms} z} . \tag{4.24}$$

In the second stage, we set $z = n\tilde{U}/P$. Because δ is oscillatory in $u \sim \ln(\tilde{U})$, the form of δ as a function of z is essentially the same as the form as a function of $\tilde{U}$. For sufficiently large n, $z \sim 1$ corresponds to small $\tilde{U}$, so the period of δ will be $\Delta\tilde{U} \sim \tilde{U}$, or $\Delta z \sim z \sim O(1)$. The contribution to the integral over z from $z \sim 1$ is thus $O(1)$, and we therefore expect $|B_n| \sim P\delta_{rms}/n$.[3] Thus, if a string initially in its ground state falls through the singularity, each mode will be excited to

$$\langle N_n \rangle \sim \frac{\delta_{rms}^2 P^2}{n^2} . \tag{4.25}$$

Let us now calculate the total excitation of the string. There are three important measures of this total excitation. The string mass is given by

$$\langle M_>^2 \rangle \sim \sum_{n=1}^{\infty} n \langle N_n \rangle , \tag{4.26}$$

while the total number of modes is given by

$$\langle N_> \rangle \sim \sum_{n=1}^{\infty} \langle N_n \rangle , \tag{4.27}$$

and the average size of the string is given by [15]

$$\langle r_> \rangle \sim \sum_{n=1}^{\infty} \frac{1}{n} \langle N_n \rangle . \tag{4.28}$$

Thus, we see that while the total number of modes and the average size of the resulting string state remain finite, the mass diverges once we pass through the plane wave. Note that the divergence doesn't come from a divergence in some individual $\langle N_n \rangle$, but rather from the sum over arbitrarily high modes, each of which makes a small contribution. The size of the individual $\langle N_n \rangle$ will depend on the size of the inhomogeneities, but the divergence occurs so long as there is any non-zero inhomogeneity. Although the

[3] The contribution from very small z will be small, as e^{-iz} will be approximately constant, and the contributions from successive half-cycles of the oscillations of δ will approximately cancel by the argument above.

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analysis was only carried out for small inhomogeneities, making the inhomogeneities larger is unlikely to soften the divergence. One might worry that this divergence is just an artifact of the plane wave approximation. That is, one might think that keeping subleading parts of the curvature would lead to a different conclusion. However, this does not seem likely, as such subleading terms describe curvature on large scales, and the string remains small compared to those scales even after it has passed through the singularity, so it should be insensitive to the behavior on such scales.[4] Thus, it seems that these solutions really are singular from the perspective of test strings.

5. Discussion

We have studied the singularity at the horizon of black strings with traveling waves discovered in [8, 9]. We have seen that quantum corrections will be important at the would-be horizon. By adding infalling matter to the solution, we obtain invariants which diverge at the horizon. The presence of large invariants is a fairly unambiguous sign that quantum corrections will become important. However, it is not clear what the effect of these quantum corrections will be. Since the classical solution predicts a finite distortion for infalling test bodies, one might suspect that their motion will not be much affected by the quantum corrections.

The quantum corrections *are* important for test objects in string theory. It is possible to describe the propagation of test strings using an approximate metric which has a plane-wave form. This approximation is valid if the test strings cross the horizon with q approximately constant (so we can ignore the q dependence in the metric), and they remain small (so we can ignore the curvature of the three-sphere). The mass-squared of a string initially in its ground state diverges as we propagate it through the singularity, due to contributions from arbitrarily high modes. Thus, it is not consistent to use a classical picture of the background spacetime in the neighbourhood of the singularity. One must replace the description given here with some kind of quantum picture once the curvature becomes sufficiently large. I have only treated the six-dimensional black string with longitudinal wave. The extension to the five-dimensional case is an essentially trivial exercise. I expect similar behaviour would be seen with internal waves as well, although it is not clear to me whether a plane-wave approximation is possible in that case.

What do these results tell us about black hole physics? The typical extreme string state will have some non-trivial momentum distribution, and hence will form a black string with singular horizon. Here we have found that we cannot evolve test strings beyond this singularity using the classical solution, but must include quantum effects. The question then becomes to what extent we can think of the appropriate quantum description as representing a black string.

[4]However, the authors of [14] argue that for plane waves which give divergent answers, changes in the solution at arbitrarily large scales will make the answers finite.

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The entropy of the classical black string solutions can be reproduced in string theory at weak coupling [6]. This appears still more mysterious in light of the present results. The presence of large curvature invariants indicates that quantum corrections to the geometry will be important, but the uncorrected geometry still gives the correct density of states. It's unclear what this means, but it may be suggesting that the full quantum solution is still in some sense like a black string. Myers has argued for the alternative point of view [16], that the presence of the singularity indicates that this state is essentially unlike a black string. The only way to obtain a black string at strong coupling is then to start with a completely thermal ensemble at weak coupling, in which only the total momentum is fixed. Thus, no information is lost in the transition from weak to strong coupling.

A fuller understanding of this puzzle will likely be useful in understanding the role of black hole and black string solutions in quantum gravity. The presence of these singularities at least gives a convincing argument that quantum effects will be important at the horizon, which is surely necessary if such effects are to have any relevance for the information loss problem. It should be borne in mind that we have only discussed the extreme solutions. It is much more difficult to discuss the effects of small changes in the metric on non-extreme black holes, as such changes will dissipate over time. Thus, the argument made here is not easy to extend to the non-extreme case.

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References

[1] G.T. Horowitz, *The origin of black hole entropy in string theory*, gr-qc/9604051.

[2] J.M. Maldacena, *Black holes in string theory*, Ph.D. thesis, Princeton University 1996, hep-th/9607235.

[3] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029].

[4] D. Garfinkle and T. Vachaspati, *Cosmic string traveling waves*, *Phys. Rev.* **D 42** (1990) 1960.

[5] D. Garfinkle, *Black string traveling waves*, *Phys. Rev.* **D 46** (1992) 4286 [gr-qc/9209002].

[6] G.T. Horowitz and D. Marolf, *Counting states of black strings with traveling waves*, *Phys. Rev.* **D 55** (1997) 835 [hep-th/9605224].

[7] G.T. Horowitz and D. Marolf, *Counting states of black strings with traveling waves. 2*, *Phys. Rev.* **D 55** (1997) 846 [hep-th/9606113].

[8] G.T. Horowitz and H. Yang, *Black strings and classical hair*, *Phys. Rev.* **D 55** (1997) 7618 [hep-th/9701077].

[9] N. Kaloper, R.C. Myers, and H. Roussel, *Wavy strings: Black or bright?* *Phys. Rev.* **D 55** (1997) 7625 [hep-th/9612248].

[10] D. Marolf, *Statistical effects and the black hole / d-brane correspondence*, *Phys. Rev.* **D 57** (1998) 2427 [hep-th/9705063].

[11] G.T. Horowitz and S.F. Ross, *Properties of naked black holes*, *Phys. Rev.* **D 57** (1998) 1098 [hep-th/9709050].

[12] G.T. Horowitz and A.R. Steif, *Strings in strong gravitational fields*, *Phys. Rev.* **D 42** (1990) 1950.

[13] G.W. Gibbons, *Quantized fields propagating in plane-wave spacetimes*, *Comm. Math. Phys.* **45** (1975) 191.

[14] H.J. de Vega and N. Sanchez, *Strings falling into space-time singularities*, *Phys. Rev.* **D 45** (1992) 2783.

[15] D. Mitchell and N. Turok, *Statistical properties of cosmic strings*, *Nucl. Phys.* **B 294** (1987) 1138.

[16] R.C. Myers, *Pure states don't wear black*, *Gen. Rel. Grav.* Gen.Rel.Grav. **29** (1997) 1217 [gr-qc/9705065].